

Anisotropic inflation reexamined: upper bound on broken rotational invariance during inflation

Atsushi Naruko,^{a,b} Eiichiro Komatsu,^{c,d} Masahide Yamaguchi^a

^aDepartment of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan

^bAPC (CNRS-Université Paris 7), 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France

^cMax-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany

^dKavli Institute for the Physics and Mathematics of the Universe (WPI), Todai Institutes for Advanced Study, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan

E-mail: naruko@th.phys.titech.ac.jp

Abstract. The presence of a light vector field coupled to a scalar field during inflation makes a distinct prediction: the observed correlation functions of the cosmic microwave background (CMB) become statistically anisotropic. We study the implications of the current bound on statistical anisotropy derived from the Planck 2013 CMB temperature data for such a model. The previous calculations based on the attractor solution indicate that the magnitude of anisotropy in the power spectrum is proportional to N^2 , where N is the number of e -folds of inflation counted from the end of inflation. In this paper, we show that the attractor solution is not compatible with the current bound, and derive new predictions using another branch of anisotropic inflation. In addition, we improve upon the calculation of the mode function of perturbations by including the leading-order slow-roll corrections. We find that the anisotropy is roughly proportional to $[2(\varepsilon_H + 4\eta_H)/3 - 4(c - 1)]^{-2}$, where ε_H and η_H are the usual slow-roll parameters and c is the parameter in the model, regardless of the form of potential of an inflaton field. The bound from Planck implies that breaking of rotational invariance during inflation (characterized by the background homogeneous shear divided by the Hubble rate) is limited to be less than $\mathcal{O}(10^{-9})$. This bound is many orders of magnitude smaller than the amplitude of breaking of time translation invariance, which is observed to be $\mathcal{O}(10^{-2})$.

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1 Introduction

Invariance of the probability distribution of primordial curvature perturbations under spatial rotation and translation is the fundamental prediction of the standard single scalar-field inflation models [1]. On the other hand, the probability distribution is only approximately invariant under spatial dilation; thus, the two-point correlation function of the primordial curvature perturbation is approximately, but not exactly, scale invariant [2]. Deviation from the exact scale invariance has been detected conclusively with more than 5 standard deviations [3, 4] which, along with stringent limits on deviation from Gaussian statistics [5, 6], supports the idea that the observed structures in the universe originate from quantum fluctuations generated during inflation.

These standard predictions depend little on details of the single-field inflation models, as the statistical properties of the probability distribution are determined primarily by symmetry of spacetime; namely, spacetime during inflation is nearly de Sitter with the Hubble expansion rate slowly varying with time, i.e., $|\dot{H}/H^2| = \mathcal{O}(10^{-2})$. Nearly, but not exactly, scale-invariant correlation function is the consequence of this. Then, the natural question is, “what if spacetime during inflation is slightly anisotropic, like a Bianchi type?” How much

do we know about violation of rotational symmetry during inflation? Can we place a bound on it?

Anisotropic inflation is a class of multi-field inflation models. They contain a vector field violating rotational symmetry (see [7–9] for reviews). As a result, N -point correlation functions of the curvature perturbation become statistically anisotropic [10–18].¹ In this paper, we shall focus on anisotropy in the two-point correlation function sourced by an F^2 term in the action. While this model has been studied extensively in the literature [11–13, 26], we show that there are subtleties missing in the literature which results in a revised prediction of this particular model. We also obtain an upper bound on violation of rotational symmetry during inflation in this model, using the latest bound from the Planck 2013 temperature anisotropy data [27].

This paper is organized as follows. In section 2, we show that the attractor solution of the F^2 model, on which the previous calculations [11–13] are based, is incompatible with the current bound on statistical anisotropy [27], and then discuss a new branch of anisotropic inflation, particularly paying attention to the background solution. In section 3, cosmological perturbations on a new background solution are discussed and their second-order actions are obtained. In section 4, we calculate the statistical anisotropy. In section 5, we discuss implications of our results for the bound on breaking of rotational invariance during inflation. We conclude in section 6. We give detailed derivation of the perturbed action in the appendix.

2 New branch in anisotropic inflation

2.1 Motivation

Watanabe, Kanno and Soda [26] found the first working model (e.g., free from a ghost) of inflation with a vector field that can produce persistent anisotropy in the background spacetime. This model provides a counter-example to the “cosmic no-hair conjecture,” which states that the spacetime rapidly approaches quasi de Sitter spacetime during inflation. The same authors found a solution of anisotropic spacetime which is an attractor (i.e., the solution that is independent of initial conditions of scalar (inflaton) and vector fields). The attractor solution yields a time-independent ratio of the energy densities of the vector and scalar fields.

However, as we shall show in this paper, the conditions in which the attractor solution is valid are inconsistent with the current observational bound on g_* [27]. The goal of this paper is to find another branch of solutions that is compatible with the observational bound, and derive new predictions of this model for statistical properties of perturbations.

The action of the model is given by [26]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu} \right], \quad (2.1)$$

where R is the Ricci scalar, $U(\phi)$ is the potential of an inflaton field, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of the vector field A_μ , and $f(\phi)$ is the coupling function between ϕ and A_μ . In this subsection, we shall follow ref. [26] and assume the following particular forms of the potential and the coupling function:

$$U(\phi) = \frac{1}{2}m^2\phi^2, \quad f(\phi) = e^{\frac{1}{2}c\phi^2}, \quad (2.2)$$

¹Anisotropic phase prior to inflation [19–21] and the so-called solid inflation [22–25] can also generate anisotropies in the correlation functions.

where m is an inflaton mass and c is a constant. Note that this form of the potential is used in this subsection only. The analysis in the later sections is completely general, and is applicable to any forms of the potential. The coupling function shall be determined by the form of the potential. Using the slow-roll approximation, $3H^2 \approx U(\phi)$ and $\ddot{\phi} \ll H\dot{\phi}$ where H is the Hubble rate, the scalar field equation can be simplified as

$$\frac{d}{d\alpha}\phi \approx -\frac{U_\phi}{U}(1 - cI), \quad d\alpha \equiv Hdt. \quad (2.3)$$

Because of the presence of the coupling $f(\phi)$, the vector field affects the equation of motion of the scalar field through a new function I defined by

$$I \equiv 2 \left(\frac{1}{2} \frac{U_\phi^2}{U^2} \right)^{-1} \frac{\rho^{\vec{A}}}{U} = \frac{I_\spadesuit}{e^{c(\phi^2 - \phi_\spadesuit^2) + 4(\alpha - \alpha_\spadesuit)}}. \quad (2.4)$$

Here, $\rho^{\vec{A}}$ is the energy density of the vector field, and the subscript \spadesuit denotes a certain epoch during inflation, $t = t_\spadesuit$.

As shown by ref. [11], I determines the amplitude of statistical anisotropy in the power spectrum of the curvature perturbation. When we write the power spectrum as [10]

$$P(\mathbf{k}) = P(k) \left[1 + g_*(\hat{\mathbf{k}} \cdot \hat{\mathbf{v}})^2 \right], \quad (2.5)$$

where $\hat{\mathbf{v}}$ is some preferred direction in space, the amplitude of statistical anisotropy, g_* , is related to I via

$$g_* = -24IN_k^2. \quad (2.6)$$

Here, N_k is the number of e -folds of inflation counted from the end of inflation to the epoch at which a perturbation with a given wavenumber, $k = |\mathbf{k}|$, left the horizon.

Solving equation (2.3), the general solution of I up to slow-roll corrections is given by

$$I = \frac{c-1}{c^2} \frac{1}{1 + \left(\frac{c-1}{c^2} \frac{1}{I_\spadesuit} - 1 \right) e^{-4(c-1)(\alpha - \alpha_\spadesuit)}}. \quad (2.7)$$

If $c-1 \geq \mathcal{O}(1)$ and I_\spadesuit is not extremely small, the second term in the denominator can be neglected, and I is simply determined by the model parameter c as

$$I \rightarrow \frac{c-1}{c^2}. \quad (2.8)$$

This is the attractor solution found by ref. [26]. Using equation (2.6), we can compare the prediction of the attractor solution for g_* directly with the observation. The current bound on g_* is given by [27] $g_* = 0.002 \pm 0.016$ (68% CL), which yields the constraint on c as

$$I = \frac{c-1}{c^2} \approx c-1 \lesssim 10^{-7} \times \left(\frac{|g_*|}{10^{-2}} \right) \left(\frac{N_k}{60} \right)^{-2}. \quad (2.9)$$

We thus find that c must be extremely close to unity. This result contradicts with the assumption made to obtain the attractor solution in the first place, i.e. $c-1 \geq \mathcal{O}(1)$. Therefore, we must conclude that the attractor solution is inconsistent with the observation.

²

² While we used the formula for g_* derived in the previous work ref. [11] to reach this conclusion, their formula needs a correction as we shall show in section 4. We have checked that the corrected formula for g_* still gives the same conclusion that the attractor solution for the background is not consistent with the observational bound on g_* .

2.2 New solution for the anisotropic background

In anisotropic inflation, the background spacetime is homogeneous but *anisotropic* because of the presence of a vector field. To describe the solution, we take the line element of Bianchi type I spacetime with the following form:

$$ds^2 = -\mathcal{N}^2(t)dt^2 + e^{2\alpha(t)} \left[e^{-4\beta(t)} dx^2 + e^{2\beta(t)} \delta_{ab} dy^a dy^b \right], \quad (2.10)$$

where e^α is the standard isotropic scale factor in an isotropic background, and e^β describes anisotropy in the expansion. The indices, a and b , take on 2 and 3 (i.e., y and z axes).

In the model of ref. [26], there exists a homogeneous vector field at the onset of inflation. Without loss of generality, we shall take the direction of the initial vector field to be the x -axis, i.e., $A_\mu = (0, u(t), 0, 0)$. One can question the origin of such a homogeneous vector field at the onset of inflation. If inflation starts, quantum fluctuations in A_μ inside the horizon would be stretched to outside the horizon, and provide a classical background $u(t)$. However, it is also natural to expect that there were fluctuating vector fields before inflation, whose gradient energy density would prevent inflation from starting. In fact, the situation is not different from the standard inflation with a scalar field. To start inflation, one must have a scalar field which is homogeneous over a few Hubble radius at the onset of inflation, but we do not know how to arrange such a homogeneous field at the beginning without fine-tuning or anthropic argument. Therefore, we shall postpone this challenging question regarding the origin of homogeneous vector and scalar fields at the onset of inflation, and proceed.

The dynamics of $u(t)$ is governed by the following equation of motion:

$$0 = \frac{1}{\sqrt{-g}} \partial_\nu \left(\sqrt{-g} f^2 F^{\mu\nu} \right) = \frac{1}{e^{3\alpha}} \partial_t \left(f^2 e^{\alpha+4\beta} \dot{u} \right) \delta^\mu_x. \quad (2.11)$$

Integrating it we obtain

$$\dot{u} = \frac{C_A}{f^2 e^{\alpha+4\beta}}, \quad (2.12)$$

with C_A being an integration constant related to the initial condition of the vector field. The energy density, $\rho^{\vec{A}}$, isotropic pressure, $P^{\vec{A}}$, and anisotropic stress, $(\pi^{\vec{A}})^i_j$, of the vector field are given by

$$\rho^{\vec{A}} = \frac{1}{2} \mathcal{V}, \quad P^{\vec{A}} = \frac{1}{6} \mathcal{V}, \quad (\pi^{\vec{A}})^x_x = -\frac{2}{3} \mathcal{V}, \quad (\pi^{\vec{A}})^a_b = \frac{1}{3} \mathcal{V} \delta^a_b, \quad (2.13)$$

where

$$\mathcal{V} \equiv \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)}} = \frac{C_A^2}{f^2 e^{4(\alpha+\beta)}}. \quad (2.14)$$

Using this function \mathcal{V} , the background Einstein equations read

$$3H^2 - 3\dot{\beta}^2 = \frac{1}{2} \dot{\phi}^2 + U(\phi) + \frac{1}{2} \mathcal{V}, \quad (2.15a)$$

$$\dot{H} + 3H^2 = U(\phi) + \frac{1}{6} \mathcal{V}, \quad (2.15b)$$

$$\ddot{\beta} + 3H\dot{\beta} = \frac{1}{3} \mathcal{V}, \quad (2.15c)$$

and the equation of motion of ϕ reads

$$\ddot{\phi} + 3H\dot{\phi} + U_\phi = \frac{f_\phi}{f} \mathcal{V}. \quad (2.16)$$

We find the background solution in the leading order of the slow-roll expansion. First, to realize anisotropic inflation, let us relate the form of the coupling function, $f(\phi)$, to an arbitrary inflaton potential, $U(\phi)$, as

$$f(\phi) = f_\bullet \exp \left[2c \int_{\phi_\bullet}^{\phi} d\phi' \frac{U}{U_\phi} \right], \quad (2.17)$$

Then the right hand side of the equation of motion of ϕ (equation (2.16)) becomes

$$\ddot{\phi} + 3H\dot{\phi} + U_\phi = 2c \frac{U}{U_\phi} \mathcal{V}. \quad (2.18)$$

The function determining the statistical anisotropy, I (equation (2.4)), now reads

$$I \equiv 2 \left(\frac{1}{2} \frac{U_\phi^2}{U^2} \right)^{-1} \frac{\rho^{\vec{A}}}{U} = \left(\frac{1}{2} \frac{U_\phi^2}{U^2} \right)^{-1} \frac{\mathcal{V}}{U}. \quad (2.19)$$

Physically, this function I is the ratio of the energy densities of the vector and scalar fields, divided by the slow-roll parameter.

So far, the basic equations are general, except for the fact that the form of $f(\phi)$ is fixed by the potential as in equation (2.17). We can, in principle, consider a case with arbitrarily strong anisotropy by solving the basic equations without any approximations. However, according to the previous formula of ref. [11], the current bound on g_* implies that I is smaller than of order 10^{-7} (equation (2.9)). While this result was obtained using the attractor solution that was not compatible with $I \ll 1$, we shall still assume that I is small, and solve the basic equations by expanding them in terms of I . Physically, we assume the ratio of the energy densities of vector and scalar fields to be smaller than the slow-roll parameter. For a small I , we find a general solution for I regardless of the form of $U(\phi)$:

$$I = I_\bullet e^{4(c-1)(\alpha-\alpha_\bullet)}, \quad I_\bullet \equiv 2 \frac{U}{U_\phi^2} \frac{C_A^2}{f^2 e^{4\alpha}} \Big|_{t=t_\bullet}. \quad (2.20)$$

where we have used the standard slow-roll conditions, $3H^2 \approx U(\phi)$ and $\ddot{\phi} \ll H\dot{\phi}$, and ignored the slow-roll terms. We shall include the leading-order slow-roll terms when studying perturbations in the next section. Note that it is the choice of the functional form of $f(\phi)$ given in equation (2.17) that enables us to obtain this solution for I .

Under the slow-roll approximation, the scalar field equation (2.16) reads

$$3H\dot{\phi} + U_\phi = cU_\phi I \quad \rightarrow \quad \frac{d\phi}{d\alpha} = -\frac{U_\phi}{U}(1 - cI) \approx -\frac{U_\phi}{U}. \quad (2.21)$$

Using this result in equation (2.17) and integrating, we obtain

$$f(\phi) = f_\bullet \exp \left[-2c \int_{\phi_\bullet}^{\phi} d\phi' \frac{d\alpha}{d\phi'} \right] = f_\bullet e^{-2c(\alpha-\alpha_\bullet)}. \quad (2.22)$$

Using this result in equation (2.20), we obtain the explicit solution for I with arbitrary $U(\phi)$. While the previous, attractor solution, $I_{\text{attractor}} = (c-1)/c^2$, was independent of time, the new solution depends on time. The parameter c now determines the rate of evolution of I , rather than the value of I itself.

3 Perturbations

3.1 Scalar and vector modes

In this section, we study cosmological perturbations to the homogeneous but anisotropic background solution obtained in the last section. Different from the Friedmann-Lemaître-Robertson-Walker (FLRW) case, where the spacetime is homogeneous *and* isotropic, our background solution has less symmetry. Our solution has only 2D rotational symmetry as opposed to 3D in the standard FLRW set-up. The perturbations can be classified based on 2D symmetry in the y - z plane; thus, they have 2D-scalar and 2D-vector types.

An arbitrary 2D-vector, v^a , can be uniquely decomposed into one scalar and one vector modes. We can extract the scalar mode from the vector by taking divergence, and then obtain the vector mode by subtracting the scalar part from the original vector. In the same way, a 2D tensor, t_{ab} , can be decomposed into two scalar and one vector modes. Therefore, all of the perturbations are classified into scalar and vector modes as follows. From the matter fields (scalar and vector fields) we have

$$\text{scalar : } \delta\phi, \delta A_0, \delta A_x, 1 \text{ of } \delta A_a, \quad \text{vector : } 1 \text{ of } \delta A_a, \quad (3.1)$$

and from the metric we have

$$\text{scalar : } \delta g_{00}, \delta g_{0x}, 1 \text{ of } \delta g_{0a}, \delta g_{xx}, 1 \text{ of } \delta g_{xa}, 2 \text{ of } \delta g_{ab}, \quad \text{vector : } 1 \text{ of } \delta g_{0a}, 1 \text{ of } \delta g_{xa}, 1 \text{ of } \delta g_{ab}, \quad (3.2)$$

In total, we have $11 (= 4 + 7)$ scalar-type and $4 (= 1 + 3)$ vector-type perturbations.

However, these 15 degrees of freedom are not independent because of general covariance and $U(1)$ gauge symmetry. From the general covariance we have $4 = 3S + 1V$ gauge degrees of freedom and $4 = 3S + 1V$ constraint equations (Hamiltonian and momentum constraints). Moreover, the $U(1)$ gauge symmetry removes $1 = 1S$ from gauge degrees of freedom and $1 = 1S$ from the constraint equation. In the end, $3 = 11 - 6 - 2$ scalar and $2 = 4 - 2$ vector perturbations are physical degrees of freedom in this system.³

In linear theory it is useful to perform Fourier decomposition since each mode evolves independently. In the FLRW case, 3D rotational symmetry enables us to take a wave vector to be in the x -direction, $\mathbf{k} = (k, 0, 0)$, for example. On the other hand, in the current setting, we only have 2D plane symmetry, which enables us to take a wave vector to be $\mathbf{k} = (k_x, k_y, 0)$, for example. Hereafter we shall drop the z -dependence of perturbations using this restricted plane symmetry. As a consequence, the scalar component of a vector v^a is stored in its y component because $\partial_a v^a = \partial_y v^y$, while the vector component can be read from the z component of v^a .

3.2 Gauge choice and classification of perturbations

As discussed in detail in [11], the most convenient gauge in studying subhorizon dynamics is the flat gauge, where information of perturbations is mostly encoded in the matter field perturbations. Here and hereafter, we shall use the same gauge. In this gauge, we write the

³In the FLRW case, they correspond to 2 tensor modes (gravitational waves), 1 scalar mode, and 2 vector modes.

metric perturbations as

$$\delta g_{\mu\nu} = \begin{pmatrix} -2A & e^{2(\alpha-2\beta)} B_x & e^{2(\alpha+\beta)} B_y & 0 \\ e^{2(\alpha-2\beta)} B_x & 2e^{2(\alpha-2\beta)} C & 0 & 0 \\ e^{2(\alpha+\beta)} B_y & 0 & 2e^{2(\alpha+\beta)} C & 0 \\ 0 & 0 & 0 & -2e^{2(\alpha+\beta)} C \end{pmatrix}, \quad (3.3)$$

and the matter perturbations as

$$\delta\phi, \quad \delta A_\mu = (\delta A_t, 0, \delta A_y, 0), \quad (3.4)$$

where we have already eliminated three perturbations in spatial components of the metric and δA_x in the matter perturbations using gauge degrees of freedom.

We insert equations (3.3) and (3.4) into the action (2.1), and expand it up to the second order in the perturbations. We give the explicit form in the appendix. Note that A , B_x , B_y , and δA_t are non-dynamical and can be integrated out because their time derivatives do not appear in the quadratic action.

3.3 Slow-roll expansion of the action

As explained in detail in the appendix, we obtain the action for canonically normalized perturbation variables after solving the constraints. We then impose the slow-roll conditions to satisfy current observational constraints such as a nearly scale-invariant power spectrum. Here, we simply summarize the action. Our action differs in detail from that used in the previous work. Bartolo et al. [13] studied a vector field coupled to inflaton in the FLRW background, while we study it in the Bianchi type I background. Watanabe et al. [11] used the Bianchi type I background, but they used the attractor solution which gives $\dot{I} = 0$. Our background, which is compatible with the current observational bound, yields \dot{I}/I of order the slow-roll parameters (see appendix B.2); thus, our action contains terms proportional to \dot{I} .

As discussed in the last subsection, there are three independent dynamical degrees of freedom. We define canonically normalized variables as

$$\delta\phi \rightarrow \varphi \equiv e^{\alpha+\beta} \delta\phi, \quad C \rightarrow \mathcal{G} \equiv \sqrt{2} e^{\alpha+\beta} C, \quad \delta A_y \rightarrow \mathcal{A} \equiv f \frac{k_x}{\tilde{k}} \delta A_y \quad (3.5)$$

with $\tilde{k} \equiv \sqrt{k_x^2 + (e^{-3\beta} k_y)^2}$. The action of the scalar perturbations reads

$$S^{\text{scalar}} = \int dt \frac{d^3 k}{(2\pi)^3} e^{\alpha-2\beta} \left(\mathcal{L}^{\varphi\varphi} + \mathcal{L}^{\mathcal{G}\mathcal{G}} + \mathcal{L}^{\mathcal{A}\mathcal{A}} + \mathcal{L}^{\varphi\mathcal{G}} + \mathcal{L}^{\varphi\mathcal{A}} + \mathcal{L}^{\mathcal{G}\mathcal{A}} \right), \quad (3.6)$$

where the first three and the last three terms correspond to auto- and cross-correlation terms, respectively. Their explicit forms are given by

$$\mathcal{L}^{\varphi\varphi} = \frac{1}{2} |\dot{\varphi}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\varphi|^2 + \frac{1}{2} H^2 \left(2 + 2\varepsilon_H + 3\eta_H + \delta m_{\varphi\varphi}^2 \right) |\varphi|^2, \quad (3.7a)$$

$$\mathcal{L}^{\mathcal{G}\mathcal{G}} = \frac{1}{2} |\dot{\mathcal{G}}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\mathcal{G}|^2 + \frac{1}{2} H^2 \left[2 - \varepsilon_H + \mathcal{O}(\varepsilon_H I) \right] |\mathcal{G}|^2, \quad (3.7b)$$

$$\mathcal{L}^{\mathcal{A}\mathcal{A}} = \frac{1}{2} |\dot{\mathcal{A}}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\mathcal{A}|^2 + \frac{1}{2} H^2 \left[2c(2c-1) + 2c \frac{4(1-c)\varepsilon_H + (1-4c)\eta_H}{3} + \delta m_{\mathcal{A}\mathcal{A}}^2 \right] |\mathcal{A}|^2, \quad (3.7c)$$

and

$$\mathcal{L}^{\varphi\mathcal{G}} = -3cH^2\sqrt{\varepsilon_H}I\sin^2\theta\varphi\mathcal{G}^* + (\text{c.c.}), \quad (3.7d)$$

$$\mathcal{L}^{\varphi\mathcal{A}} = \sqrt{6}cH\sqrt{I}\sin\theta(\dot{\varphi} - H\varphi)\mathcal{A}^* + (\text{c.c.}), \quad (3.7e)$$

$$\mathcal{L}^{\mathcal{G}\mathcal{A}} = -\sqrt{\frac{3}{2}}H\sqrt{\varepsilon_H}I\sin\theta(\dot{\mathcal{G}} - H\mathcal{G})\mathcal{A}^* + (\text{c.c.}), \quad (3.7f)$$

with

$$\sin\theta \equiv \frac{e^{-3\beta}k_y}{\tilde{k}}, \quad (3.8a)$$

$$\delta m_{\varphi\varphi}^2 \equiv 2\left[12c^2\sin^2\theta - (4c^2 + c + 1)\right]I + \mathcal{O}(\varepsilon_H I), \quad (3.8b)$$

$$\delta m_{\mathcal{A}\mathcal{A}}^2 \equiv -2c(c + 1)I + \mathcal{O}(\varepsilon_H I). \quad (3.8c)$$

Here and hereafter, the momentum dependence of the variables is abbreviated. The slow-roll parameters are defined as

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2}, \quad \eta_H \equiv \frac{\ddot{H}}{2H\dot{H}}. \quad (3.9)$$

Equation (3.7c) shows that the effective mass squared of \mathcal{A} is $m_{\text{eff}}^2 = \frac{1}{2}H^2\left[2c(2c - 1) + \mathcal{O}(\varepsilon)\right]$, which is quadratic in c . Therefore, two different c can reproduce the same spectrum of \mathcal{A} . For example, the mass term vanishes for $c = 0$ because $f(\phi)$ reduces to unity and the action simply becomes $F_{\mu\nu}F^{\mu\nu}$. The mass term vanishes also for $c = 1/2$ at leading order in the slow-roll expansion. This can be understood as follows. The action for a homogeneous vector field in the FLRW universe can be rewritten by introducing a canonical vector field $\vec{B} = f\vec{A}$ as

$$\begin{aligned} -\frac{1}{4}\int d^4x\sqrt{-g}f^2F_{\mu\nu}F^{\mu\nu} &\propto \int d^4x e^\alpha f^2 \dot{\vec{A}}^2 \\ &= \int d^4x e^\alpha \left\{ \dot{\vec{B}}^2 + \left[\frac{1}{fe^\alpha} \frac{d}{dt}(fe^\alpha) \right] \vec{B}^2 \right\}. \end{aligned} \quad (3.10)$$

For $c = 1/2$, $f \propto e^{-2c\alpha} = e^{-\alpha}$; thus, the second term becomes

$$\frac{1}{fe^\alpha} \frac{d}{dt}(fe^\alpha) = -\dot{H} = \varepsilon_H H^2, \quad (3.11)$$

i.e., suppressed by the slow-roll parameter.

4 Statistical anisotropy

4.1 g_* : general potential

In this section, we shall investigate the statistical properties of primordial fluctuations generated during anisotropic inflation. To this end, we use the so-called in-in formalism. In the interaction picture, the interaction Hamiltonian is given in terms of $\mathcal{L}^{\varphi\mathcal{G}}$, $\mathcal{L}^{\varphi\mathcal{A}}$, and $\mathcal{L}^{\mathcal{G}\mathcal{A}}$ as

$$H_{\text{int}}^{\varphi\mathcal{G}} \equiv \int d^3k (-e^{\alpha-2\beta}\mathcal{L}^{\varphi\mathcal{G}}), \quad H_{\text{int}}^{\varphi\mathcal{A}} \equiv \int d^3k (-e^{\alpha-2\beta}\mathcal{L}^{\varphi\mathcal{A}}), \quad H_{\text{int}}^{\mathcal{G}\mathcal{A}} \equiv \int d^3k (-e^{\alpha-2\beta}\mathcal{L}^{\mathcal{G}\mathcal{A}}). \quad (4.1)$$

The dominant correction to the power spectrum comes from $H_{\text{int}}^{\varphi\mathcal{A}}$.

We define mode functions and annihilation/creation operators of perturbations, \mathcal{Q}^λ ($\lambda = \varphi, \mathcal{G}, \mathcal{A}$), as

$$\mathcal{Q}_{\mathbf{k}}^\lambda(\tau) = u_{\mathbf{k}}^\lambda(\tau)a_{\mathbf{k}}^\lambda + u_{-\mathbf{k}}^{\lambda*}(\tau)a_{-\mathbf{k}}^{\lambda\dagger}, \quad (4.2)$$

where τ is the conformal time defined as $dt = e^{\alpha-2\beta}d\tau$ and u^λ is the solution of the following equation:

$$\partial_\tau^2 u_{\mathbf{k}}^\lambda + \left(k^2 - \frac{\nu_\lambda^2 - 1/4}{\tau^2}\right) u_{\mathbf{k}}^\lambda = 0, \quad (4.3)$$

with ν_λ being

$$\nu_\varphi \equiv \frac{3}{2} + 2\varepsilon_H + \eta_H + \mathcal{O}(\varepsilon_H^2, I), \quad (4.4a)$$

$$\nu_{\mathcal{G}} \equiv \frac{3}{2} + \varepsilon_H + \mathcal{O}(\varepsilon_H^2, I), \quad (4.4b)$$

$$\nu_{\mathcal{A}} \equiv \frac{3}{2} + \frac{2}{3}[2\varepsilon_H - \eta_H + 3(c-1)] + \mathcal{O}(\varepsilon_H^2, I, (c-1)^2). \quad (4.4c)$$

We have also used the relation $a = (-\tilde{k}\tau)^{-(1+\varepsilon_H)}$, which is valid in the quasi de Sitter approximation; that is to say, the time dependence of ε_H is negligible. Using the Bunch-Davies initial condition, each mode function is given in terms of the Hankel function, $H_\nu(x)$, as

$$u_{\mathbf{k}}^\lambda = C^\lambda \sqrt{-\tau} H_{\nu_\lambda}^{(1)}(-\tilde{k}\tau), \quad C^\lambda = \frac{\sqrt{\pi}}{2} \exp\left[i\left(\frac{1}{2}\nu_\lambda + \frac{1}{4}\right)\pi\right]. \quad (4.5)$$

The power spectrum calculated from the above mode functions must be consistent with the nearly scale-invariant spectrum, i.e., $\nu - 3/2 = \mathcal{O}(\varepsilon_H, \eta_H)$. This demands I and its time-variation be smaller than of order the slow-roll parameters:

$$I(t) \leq \mathcal{O}(\varepsilon_H), \quad \left|\frac{\dot{I}}{HI}\right| \leq \mathcal{O}(\varepsilon_H) \iff |c-1| \leq \mathcal{O}(\varepsilon_H). \quad (4.6)$$

The first condition is consistent with our starting assumption that I is small and we can expand the equations in terms of I . The mode function given in equation (4.5) is an approximate solution, which is valid so long as ν_λ is independent of time. While the variation of the slow-roll parameters is small, ν_λ may be regarded as constant only during some number of e -folds, ΔN , which is typically of order or less than $1/\varepsilon_H$. This, however, does not necessarily coincide with the number of e -folds from the horizon exit to the end of inflation. Thus, when we estimate the evolution of the perturbations on super-horizon scales, the use of this mode function can be justified only for some number e -folds after the horizon exit. If we were to calculate the full evolution of the perturbations on super-horizon scales, we would need to perform numerical calculations without using the Hankel functions as approximate solutions (which is beyond the scope of this paper). Instead, in the following analysis, we shall use the Hankel functions as the modes functions. Thus, our solutions are valid only until some epoch, τ_ε , which is of order or less than $1/\varepsilon_H$ e -folds after the horizon exit.

Now we are in the position to calculate g_* . We find

$$\frac{\langle \text{in} | \varphi_{\mathbf{k}} \varphi_{\mathbf{p}} | \text{in} \rangle}{\langle 0 | \varphi_{\mathbf{k}} \varphi_{\mathbf{p}} | 0 \rangle} \supset 24 \sin^2 \theta \frac{|C^\varphi|^2 |C^\mathcal{A}|^2 \Gamma^2(\nu^\mathcal{A})}{(\sqrt{\pi}/2)^4 \Gamma^2(\nu^\varphi)} \left[\int_{\tau_l}^{\tau_\varepsilon} d\tau a(\tau) H(\tau) \sqrt{I(\tau)} \left(\frac{-k\tau}{2} \right)^{\nu^\varphi - \nu^\mathcal{A}} \right]^2, \quad (4.7)$$

where τ_l is a certain epoch shortly after the horizon exit, but well before τ_ε at which the Hankel function ceases to be a good approximation. The contribution from the subhorizon regime, $\tau < \tau_l$ is negligible because the integrand oscillates rapidly and does not contribute to the integral. Using the asymptotic form of $H_\nu^{(1)}(x)$ in the $x \rightarrow 0$ limit,

$$H_\nu^{(1)}(x) \rightarrow -\frac{i}{\pi} \left(\frac{x}{2} \right)^{-\nu} \left[\Gamma(\nu) + \mathcal{O}(x^2) \right] + \left(\frac{x}{2} \right)^\nu \left[\frac{1}{\Gamma(\nu+1)} + \mathcal{O}(x^2) \right], \quad (4.8)$$

and integrating, we obtain

$$g_* \approx -24 I_\spadesuit \left[\frac{(-k\tau_\varepsilon)^\delta - (-k\tau_l)^\delta}{2^{\nu_\varphi - \nu_\mathcal{A}} \delta} \right]^2 (-k\tau_\spadesuit)^{4(c-1) - 2(\varepsilon_H + \eta_H)}, \quad (4.9)$$

with

$$\delta \equiv \nu^\varphi - \nu^\mathcal{A} + \eta_H - 2(c-1) = 2 \left[\frac{\varepsilon_H + 4\eta_H}{3} - 2(c-1) \right]. \quad (4.10)$$

This expression for g_* is one of the main results of this paper.

If we choose τ_\spadesuit and τ_l to be at the horizon exit, τ_* , defined by $-k\tau_* = 1$ for each mode \mathbf{k} , the expression simplifies to

$$g_* \approx -24 I_* \left[\frac{(-k\tau_\varepsilon)^\delta - 1}{2^{\nu_\varphi - \nu_\mathcal{A}} \delta} \right]^2. \quad (4.11)$$

But it should be noted that the final result given above still depends on the choice of τ_ε .

Our results on g_* in the new, non-attractor branch of anisotropic inflation with the leading-order slow-roll terms are qualitatively different from the previous result given in equation (2.6), which was based on the attractor branch and the de Sitter mode function. Our results do not depend on the number of e -folds counted from the end of inflation, N^2 , but on $1/\delta^2$. Moreover we have an additional contribution from the numerator in equation (4.11), which is $[(-k\tau_\varepsilon)^\delta - 1]^2$. This main difference comes from the precise estimation of the integral. By choosing the time coordinates as N ($dN = -He^\alpha d\tau$) in stead of τ , we obtain

$$\int_{\tau_*}^{\tau_\varepsilon} d\tau (-k\tau)^{-1+\delta} \propto \int_{N_\varepsilon}^{N_*} dN e^{-N\delta}. \quad (4.12)$$

As $N \approx 60$ and $\delta = \mathcal{O}(10^{-2})$, the exponent, $N\delta$, is of order unity, and thus we cannot expand it. This is the main reason why we have obtained a different formula from ref. [11] besides the difference on the background solution. Of course, if δ accidentally becomes much smaller than of order the slow-roll parameter ($\delta \ll 10^{-2}$), we can expand the exponential factor and the final result may be proportional to $(\Delta N)^2 = (N_* - N_\varepsilon)^2$. Even in this case, however, $\Delta N = N_* - N_\varepsilon$ may not be the same as the number of e -folds counted from the end of inflation.

4.2 g_* : $U(\phi) \propto \phi^n$

For slow-roll inflation with a monomial potential, $U(\phi) \propto \phi^n$, the slow-roll parameters can be expressed in terms of the number of e -folds. Then, it is possible to write g_* in terms of N_k^2 , similar to the previous result given in equation (2.6). However, the resulting g_* is different from equation (2.6) quantitatively, as the newly-found factor in the numerator, $[(-k\tau_\varepsilon)^\delta - 1]^2 = (1 - e^{-\delta\Delta N})^2$, gives an additional correction which is not necessarily of order unity, as shown below.

For $U(\phi) \propto \phi^n$, the slow-roll parameters are given by

$$\varepsilon_H = \frac{n}{4N_k} + \mathcal{O}(\varepsilon_H^2), \quad \eta_H = \frac{2-n}{4N_k} + \mathcal{O}(\varepsilon_H^2). \quad (4.13)$$

Substituting these into the expression of g_* , we obtain

$$g_* = -24I_* N_k^2 \times 2^{-\frac{10-3n}{6N_k}} \left[\frac{1 - e^{-\frac{8-3n}{6} \frac{\Delta N}{N_k} + 4(c-1)\Delta N}}{\frac{8-3n}{6} - 4(c-1)N_k} \right]^2. \quad (4.14)$$

Assuming $\delta\Delta N \ll 1$, this further reduces to

$$g_* \simeq -24I_* N_k^2 \times 2^{-\frac{10-3n}{6N_k}} \left(\frac{\Delta N}{N_k} \right)^2. \quad (4.15)$$

As is clear from its expression, the dependence on N_k^2 can be recovered. However, at the same time, the additional correction term, $2^{-\frac{10-3n}{6N_k}} (\Delta N/N_k)^2$ appears.

5 Bound on broken rotational invariance during inflation

De Sitter spacetime has ten isometries. Inflation occurs in quasi de Sitter space, in which time-translation invariance is broken softly by of order $\varepsilon_H = -\dot{H}/H^2 = \mathcal{O}(10^{-2})$. Broken time-translation invariance implies broken dilation invariance, which leads to a departure from the scale-invariant power spectrum of curvature perturbations of order ε_H . This has been detected convincingly by WMAP [3] and Planck [4].

The other nine isometries correspond to three spatial translation and three rotation, as well as three isometries resulting in the special conformal transformation in the infinite future [28]. The latter three isometries are broken during inflation, while the former six isometries (spatial translation and rotation) are respected by single-field inflation. Testing these isometries by the cosmological data thus offers a powerful probe of the physics of inflation. In this paper, we focus on testing rotational invariance.

In the absence of anisotropic stress, the dimensionless shear due to anisotropic expansion, $\dot{\beta}/H$, decays as a^{-3} because the Einstein equation gives $\dot{\beta} + 3H\dot{\beta} = 0$ (see equation (2.15c)). If the universe had the shear of order unity before inflation, and the total number of e -folds of inflation was just enough to solve the flatness problem, the natural value for the residual shear is $|\dot{\beta}|/H = e^{-3N} \approx e^{-180}$.

On the other hand, Maleknejad and Sheikh-Jabbari [29] showed that it is possible to generate $\dot{\beta}/H$ of order ε_H or smaller while not spoiling slow-roll inflation, if there is a persisting anisotropic stress during inflation. This is the scenario that we studied in this paper. Therefore, without fine-tuning, we would expect $\dot{\beta}/H$ to be either e^{-180} or 10^{-2} .

Using $I \approx \mathcal{V}/(\varepsilon_H U)$, $|\dot{\beta}|/H \approx \mathcal{V}/(3U)$, and $g_* \approx -24I\varepsilon_H^{-2}$ (up to corrections of order unity), we find $g_* \approx -72\varepsilon_H^{-3}(|\dot{\beta}|/H)$. Therefore, we would expect $|g_*|$ to be either 10^{-70} or 10^6 for $\varepsilon_H \approx 10^{-2}$. The latter is excluded by the Planck data, which gives the 95% bound of $|g_*| < 0.03$. Reversing the argument, we find a bound on the shear during inflation of order

$$\frac{|\dot{\beta}|}{H} < 10^{-9}. \quad (5.1)$$

This bound is another main result of this paper. This result relies on the specific model of anisotropic inflation connecting g_* to the background spacetime. Generality of this bound is not known at the moment, and it merits further study, perhaps in the language of Effective Field Theory (EFT) of inflation [30, 31]. In the nominal case of single-field inflation, a Nambu-Goldstone boson associated with breaking of time-translation invariance gives the curvature perturbation. In the scenario we studied in this paper, spatial-rotation invariance is broken such that $O(3) \rightarrow O(2)$. This would generate two additional Nambu-Goldstone bosons, disguising themselves as, e.g., anisotropic stresses, $(\pi^{\vec{A}})^x_x$ and $(\pi^{\vec{A}})^y_y$. Developing EFT for this case would shed light on generality of the connection between g_* and rotational symmetry of the background spacetime.

6 Conclusion

In this paper, we have used the observational bound on statistical anisotropy in the power spectrum parametrized by g_* [27] to infer the bound on broken rotational invariance in the background spacetime during inflation. We found a rather stringent (albeit model-dependent) bound, $|\dot{\beta}|/H < 10^{-9}$, whereas the natural expectation would be either $e^{-3N} \approx e^{-180}$ or $\varepsilon_H \approx 10^{-2}$.

To derive this bound, we uncovered two subtleties missing in the previous calculations of g_* from anisotropic inflation with $f^2(\phi)F^2$ in the action [26]. One is that the attractor solution giving $I = (c-1)/c^2$ is not compatible with the observational bound on g_* , and thus another branch must be used. The other is that the leading slow-roll corrections to the de Sitter mode function changes the prediction for g_* qualitatively, by replacing $g_* \propto N_k^2$ with $[2(\varepsilon_H + 4\eta_H)/3 - 4(c-1)]^{-2}$. For large-field inflation models with a monomial potential, our result gives an order-unity correction to the previous formula for g_* , i.e., $g_* = -\mathcal{O}(1) \times 24IN_k^2$, because $N_k = \mathcal{O}(1)/\varepsilon_H$. For small-field models, the correction can be larger.

The connection between g_* and the background spacetime depends on models, and generality of our result is still unclear. To address this, EFT of inflation breaking spatial rotation symmetry seems a promising avenue.

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A Detailed derivation of the reduced action

In this appendix, we explain in detail how to obtain the reduced action for the canonical variables under the slow-roll approximation. Essential steps are :

- integrate out non-dynamical variables, such as the lapse function and shift vector.
- introduce canonical variables so as to normalize the kinetic term
- perform the slow-roll expansion

A.1 Definitions and expansion of the action

First we summarize the definition of perturbations. The metric perturbations are given by

$$\delta g_{\mu\nu} = \begin{pmatrix} -2A & e^{2(\alpha-2\beta)} B_x & e^{2(\alpha+\beta)} B_y & 0 \\ e^{2(\alpha-2\beta)} B_x & 2e^{2(\alpha-2\beta)} C & 0 & 0 \\ e^{2(\alpha+\beta)} B_y & 0 & 2e^{2(\alpha+\beta)} C & 0 \\ 0 & 0 & 0 & -2e^{2(\alpha+\beta)} C \end{pmatrix}, \quad (\text{A.1})$$

and matter perturbations are

$$\delta\phi, \quad \delta A_\mu = (\delta A_t, 0, \delta A_y, 0). \quad (\text{A.2})$$

After substituting the above perturbations into the action and expanding it up to second order in perturbations, one obtains

$$\begin{aligned} S^{\text{scalar}} &= \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]^{\text{scalar}} \\ &= \int d^4x e^{3\alpha} \left\{ -3 \left(A^2 - AC + \frac{1}{2} C^2 \right) (H^2 - \dot{\beta}^2) \right. \\ &\quad - (A - C) \left[(2H - \dot{\beta}) B_{y,y} - 2(H + \dot{\beta})(\dot{C} - B_{x,x}) \right] + 2(H + \dot{\beta})(B_x C_{,x} + B_y C_{,y}) \\ &\quad + \frac{1}{4} \left(e^{-6\beta} B_{x,y}^2 + 2B_{x,y} B_{y,x} - 4B_{x,x} B_{y,y} + e^{6\beta} B_{y,x}^2 \right) - e^{-2(\alpha-2\beta)} C_{,x}^2 - e^{-2(\alpha+\beta)} C_{,y}^2 + \dot{C}^2 \\ &\quad + \frac{1}{2} \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)}} \left[\left(\frac{f_{\phi\phi}}{f} + \frac{f_\phi^2}{f^2} \right) \delta\phi^2 - 2 \frac{f_\phi}{f} \delta\phi (A + C) + A^2 + AC + \frac{1}{2} C^2 \right] \\ &\quad + \frac{f^2 \dot{u}}{e^{2(\alpha-2\beta)}} \left[- \left(2 \frac{f_\phi}{f} \delta\phi - A - C \right) \delta A_{t,x} + \delta A_{y,x} B_y \right] \\ &\quad + \frac{1}{2} \frac{f^2}{e^{2(\alpha-2\beta)}} \left[\delta A_{t,x}^2 + e^{-6\beta} (\delta A_y - \delta A_{t,y})^2 - e^{-2(\alpha+\beta)} \delta A_{y,x}^2 \right] \\ &\quad + \frac{1}{2} \left[\dot{\phi}^2 - 2\dot{\phi}(A - C)\delta\phi + \dot{\phi}^2 \left(A^2 - AC + \frac{C^2}{2} \right) \right] - C \left(A + \frac{C}{2} \right) U - (A + C) U_\phi \delta\phi \\ &\quad \left. - \dot{\phi} (B_x \delta\phi_{,x} + B_y \delta\phi_{,y}) - \frac{1}{2} \left(e^{-2(\alpha-2\beta)} \delta\phi_{,x}^2 + e^{-2(\alpha+\beta)} \delta\phi_{,y}^2 \right) - \frac{1}{2} U_{\phi\phi} \delta\phi^2 \right\}. \quad (\text{A.3}) \end{aligned}$$

A.2 Solving constraints

Since A , B_x , B_y , and δA_t are non-dynamical (their time derivatives do not appear in the action), they can be integrated out. The terms related to δA_t are given by

$$S^{\text{scalar}} \supset \int d^4x e^\alpha f^2 \left[-e^{4\beta} \left(2 \frac{f_\phi}{f} \delta\phi - A - C \right) \dot{u} \delta A_{t,x} + \frac{1}{2} e^{4\beta} \delta A_{t,x}^2 + \frac{1}{2} e^{-2\beta} (\delta A_{t,y}^2 - 2 \delta \dot{A}_y \delta A_{t,y}) \right]. \quad (\text{A.4})$$

Completing the square yields

$$\int dt \frac{d^3k}{(2\pi)^3} e^{3\alpha} \left[\frac{1}{2} \left| \delta A_t + i \frac{k_x}{\tilde{k}^2} \left(2 \frac{f_\phi}{f} \delta\phi - A - C \right) \dot{u} + i \frac{k_y}{\tilde{k}^2} e^{-6\beta} \delta \dot{A}_y \right|^2 - \frac{1}{2} \left| \frac{k_x}{\tilde{k}^2} \left(2 \frac{f_\phi}{f} \delta\phi - A - C \right) \dot{u} + \frac{k_y}{\tilde{k}^2} e^{-6\beta} \delta \dot{A}_y \right|^2 \right], \quad (\text{A.5})$$

where we have moved to Fourier space and defined \tilde{k} as

$$\tilde{k} \equiv \sqrt{k_x^2 + (e^{-3\beta} k_y)^2}. \quad (\text{A.6})$$

Note that each perturbation variable depends on momenta, although we do not write it explicitly.

In a similar way, the terms containing B_x are given by

$$\int d^4x e^{3\alpha} \left[2(H + \dot{\beta}) (B_x C_{,x} - (A - C) B_{x,x}) + \frac{1}{4} (e^{-6\beta} B_{x,y}^2 + 2 B_{x,y} B_{y,x} - 4 B_{x,x} B_{y,y}) - \dot{\phi} B_x \delta\phi \right], \quad (\text{A.7})$$

which can be rewritten as

$$\int dt \frac{d^3k}{(2\pi)^3} e^{3\alpha} \left[e^{-6\beta} k_y^2 \left| \frac{1}{2} B_x + \frac{k_x}{k_y^2} e^{6\beta} \left(i 2(H + \dot{\beta}) A - \frac{1}{2} k_y B_y - i \dot{\phi} \delta\phi \right) \right|^2 - \frac{1}{4} k_x^2 e^{6\beta} \left| B_y - i \frac{2}{k_y} (2(H + \dot{\beta}) A - \dot{\phi} \delta\phi) \right|^2 \right]. \quad (\text{A.8})$$

As for B_y , we find that B_y appears only linearly in the action:

$$\int dt \frac{d^3k}{(2\pi)^3} e^{3\alpha} i k_y B_y^* \left[\left(2H - \dot{\beta} + 2(H + \dot{\beta}) \frac{k_x^2}{k_y^2} e^{6\beta} \right) A + 3\dot{\beta} C + \frac{f^2}{e^{2(\alpha-2\beta)}} \frac{k_x}{k_y} \dot{u} \delta A_y - \left(1 + \frac{k_x^2}{k_y^2} e^{6\beta} \right) \dot{\phi} \delta\phi \right], \quad (\text{A.9})$$

and thus B_y behaves like a Lagrange multiplier. Variation with respect to B_y allows us to express A in terms of the other dynamical variables as

$$A = \frac{1}{\lambda} \left(-3\dot{\beta} C - \frac{f^2}{e^{2(\alpha-2\beta)}} \frac{k_x}{k_y} \dot{u} \delta A_y + \frac{\tilde{k}^2}{k_y^2} e^{6\beta} \dot{\phi} \delta\phi \right) \quad (\text{A.10})$$

with

$$\lambda \equiv H - 2\dot{\beta} + (H + \dot{\beta}) \left(1 + 2 \frac{k_x^2}{k_y^2} e^{6\beta} \right). \quad (\text{A.11})$$

It is now manifest that the action can be written only in terms of the dynamical variables C , $\delta\phi$, and δA_y . The kinetic terms in the action take the following form,

$$S^{\text{scalar}} \supset \int dt \frac{dk^3}{(2\pi)^3} e^{\alpha-2\beta} \left(e^{2(\alpha+\beta)} |\dot{C}|^2 + \frac{1}{2} e^{2(\alpha+\beta)} |\dot{\delta\phi}|^2 + \frac{1}{2} f^2 \frac{k_x^2}{\tilde{k}^2} |\delta\dot{A}_y|^2 \right), \quad (\text{A.12})$$

which enables us to define canonically normalized variables as

$$C \rightarrow \mathcal{G} \equiv \sqrt{2} e^{\alpha+\beta} C, \quad \phi \rightarrow \varphi \equiv e^{\alpha+\beta} \delta\phi, \quad \delta A_y \rightarrow \mathcal{A} \equiv f \frac{k_x}{\tilde{k}} \delta A_y. \quad (\text{A.13})$$

A.3 Action written with canonical variables

We shall write down the reduced action in terms of the canonical variables \mathcal{G} , φ , and \mathcal{A} .

$$S^{\text{scalar}} = \int dt \frac{dk^3}{(2\pi)^3} e^{\alpha-2\beta} \left(\mathcal{L}^{\mathcal{G}\mathcal{G}} + \mathcal{L}^{\varphi\varphi} + \mathcal{L}^{\mathcal{A}\mathcal{A}} + \mathcal{L}^{\mathcal{G}\varphi} + \mathcal{L}^{\varphi\mathcal{A}} + \mathcal{L}^{\mathcal{A}\mathcal{G}} \right), \quad (\text{A.14})$$

where

$$\begin{aligned} \mathcal{L}^{\mathcal{G}\mathcal{G}} = & \frac{1}{2} |\dot{\mathcal{G}}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\mathcal{G}|^2 + \frac{1}{2} \left\{ \frac{1}{e^{\alpha-2\beta}} \frac{d}{dt} \left[e^{\alpha-2\beta} (H + \dot{\beta}) \left(2 + 3 \frac{\dot{\beta}}{\lambda} \right) \right] + 3(H + \dot{\beta})^2 \left(1 + 2 \frac{\dot{\beta}}{\lambda} \right) \right\} |\mathcal{G}|^2 \\ & - \frac{1}{2} \left\{ \left(1 + 9 \frac{\dot{\beta}^2}{\lambda^2} \right) U + 36 e^{6\beta} \frac{k_x^2}{k_y^2} (H + \dot{\beta})^2 \frac{\dot{\beta}^2}{\lambda^2} + \frac{f^2 \dot{u}^2}{2 e^{2(\alpha-2\beta)}} \left[\frac{k_x^2}{\tilde{k}^2} \left(1 - 3 \frac{\dot{\beta}}{\lambda} \right)^2 + 6 \frac{\dot{\beta}}{\lambda} \right] \right\} |\mathcal{G}|^2, \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} \mathcal{L}^{\varphi\varphi} = & \frac{1}{2} |\dot{\varphi}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\varphi|^2 + \frac{1}{2} \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)}} \left[\frac{f\phi\phi}{f} + \frac{f_\phi^2}{f^2} - 2 \frac{f\phi}{f} e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}}{\lambda} - \frac{k_x^2}{\tilde{k}^2} \left(2 \frac{f\phi}{f} - e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}}{\lambda} \right)^2 \right] |\varphi|^2 \\ & + \frac{1}{2} \left[(2H - \dot{\beta})(H + \dot{\beta}) + 3H^2 e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}^2}{\lambda} + \dot{H} + \ddot{\beta} + e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}\ddot{\phi}}{\lambda} + \dot{\phi} \frac{d}{dt} \left(e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}}{\lambda} \right) \right] |\varphi|^2 \\ & - \left[\left(e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}}{\lambda} \right)^2 U + e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{\dot{\phi}}{\lambda} U_\phi + \frac{1}{2} U_{\phi\phi} + e^{6\beta} \frac{k_x^2}{k_y^2} \dot{\phi}^2 \left(1 - 2 e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{H + \dot{\beta}}{\lambda} \right)^2 \right] |\varphi|^2, \end{aligned} \quad (\text{A.16})$$

$$\begin{aligned} \mathcal{L}^{\mathcal{A}\mathcal{A}} = & \frac{1}{2} |\dot{\mathcal{A}}|^2 - \frac{1}{2} \frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}} |\mathcal{A}|^2 + \frac{1}{2} \left\{ \frac{1}{e^{\alpha-2\beta}} \frac{d}{dt} \left[e^{\alpha-2\beta} \left(\frac{\dot{f}}{f} - \frac{\dot{\tilde{k}}}{\tilde{k}} + \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)} \lambda} \right) \right] + \left(\frac{\dot{f}}{f} - \frac{\dot{\tilde{k}}}{\tilde{k}} \right)^2 \right\} |\mathcal{A}|^2 \\ & + \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)} \lambda} \left(\frac{\dot{f}}{f} - \frac{\dot{\tilde{k}}}{\tilde{k}} - 4 e^{12\beta} \frac{k_x^2 \tilde{k}^2}{k_y^4} \frac{(H + \dot{\beta})^2}{\lambda} - e^{6\beta} \frac{\tilde{k}^2}{k_y^2} \frac{U}{\lambda} - \frac{1}{2} \frac{f^2 \dot{u}^2}{e^{2(\alpha-2\beta)} \lambda} e^{6\beta} \frac{k_x^2}{k_y^2} \right) |\mathcal{A}|^2, \end{aligned} \quad (\text{A.17})$$

and

$$\begin{aligned}
2\mathcal{L}^{\mathcal{G}\varphi} = & \sqrt{2}(H + \dot{\beta})e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{\dot{\phi}}{\lambda}\varphi\dot{\mathcal{G}}^* + \frac{1}{\sqrt{2}}\left(1 + 3\frac{\dot{\beta}}{\lambda}\right)\dot{\phi}\dot{\varphi}\mathcal{G}^* \\
& - \frac{1}{\sqrt{2}}\frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}}\left(1 - 3\frac{\dot{\beta}}{\lambda}\right)\left[\frac{f_\phi}{f} - \frac{k_x^2}{\tilde{k}^2}\left(2\frac{f_\phi}{f} - e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{\dot{\phi}}{\lambda}\right)\right]\varphi\mathcal{G}^* \\
& - \frac{1}{\sqrt{2}}\left[\left(1 + 3\frac{\dot{\beta}}{\lambda}\right)\dot{\phi}(H + \dot{\beta}) + \left(2(H + \dot{\beta})^2 - 6\frac{\dot{\beta}}{\lambda}U - \frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}}\right)e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{\dot{\phi}}{\lambda}\right]\varphi\mathcal{G}^* \\
& - \frac{1}{\sqrt{2}}\left[\left(1 - 3\frac{\dot{\beta}}{\lambda}\right)U_\phi + 12e^{6\beta}\frac{k_x^2}{k_y^2}(H + \dot{\beta})\frac{\dot{\beta}}{\lambda}\dot{\phi}\left(1 - 2e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{H + \dot{\beta}}{\lambda}\right)\right]\varphi\mathcal{G}^* + (c.c.),
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
2\mathcal{L}^{\varphi\mathcal{A}} = & \frac{f\dot{u}}{e^{\alpha-2\beta}\lambda}e^{3\beta}\frac{\tilde{k}}{k_y}\left[\dot{\phi}\dot{\varphi}\mathcal{A}^* - e^{-6\beta}\frac{k_y^2}{\tilde{k}^2}\left(2\lambda\frac{f_\phi}{f} - e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\dot{\phi}\right)\varphi\mathcal{A}^* + \left(2e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{\dot{\phi}}{\lambda}U - \dot{\phi}(H + \dot{\beta}) + U_\phi\right)\varphi\mathcal{A}^*\right] \\
& + \frac{f\dot{u}}{e^{\alpha-2\beta}}e^{3\beta}\frac{\tilde{k}}{k_y}\left[-4e^{6\beta}\frac{k_x^2}{k_y^2}\frac{(H + \dot{\beta})\dot{\phi}}{\lambda}\left(1 - 2e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{H + \dot{\beta}}{\lambda}\right) + \frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}\lambda}\frac{f_\phi}{f}\right]\varphi\mathcal{A}^* \\
& + \frac{f\dot{u}}{e^{\alpha-2\beta}}e^{3\beta}\frac{\tilde{k}}{k_y}\left[e^{-6\beta}\frac{k_y^2}{\tilde{k}^2}\left(\frac{\dot{f}}{f} - \frac{\dot{\tilde{k}}}{\tilde{k}}\right) - \frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}\lambda}\frac{k_x^2}{\tilde{k}^2}\right]\left(2\frac{f_\phi}{f} - e^{6\beta}\frac{\tilde{k}^2}{k_y^2}\frac{\dot{\phi}}{\lambda}\right)\varphi\mathcal{A}^* + (c.c.),
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
2\mathcal{L}^{\mathcal{A}\mathcal{G}} = & \frac{f\dot{u}}{e^{\alpha-2\beta}}e^{3\beta}\frac{\tilde{k}}{k_y}\left[\frac{1}{\sqrt{2}}e^{-6\beta}\frac{k_y^2}{\tilde{k}^2}\left(1 - 3\frac{\dot{\beta}}{\lambda}\right)\mathcal{G}\dot{\mathcal{A}}^* - \sqrt{2}\frac{H + \dot{\beta}}{\lambda}\dot{\mathcal{G}}\mathcal{A}^*\right] \\
& + \sqrt{2}\frac{f\dot{u}}{e^{\alpha-2\beta}\lambda}e^{3\beta}\frac{\tilde{k}}{k_y}\left[(H + \dot{\beta})^2\left(1 - 12e^{6\beta}\frac{k_x^2}{k_y^2}\frac{\dot{\beta}}{\lambda}\right) - 3\frac{\dot{\beta}}{\lambda}U - \frac{1}{2}\frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}}\right]\mathcal{G}\mathcal{A}^* \\
& - \frac{1}{\sqrt{2}}\frac{f\dot{u}}{e^{\alpha-2\beta}}e^{-3\beta}\frac{k_y}{\tilde{k}}\left(1 - 3\frac{\dot{\beta}}{\lambda}\right)\left(\frac{\dot{f}}{f} - \frac{\dot{\tilde{k}}}{\tilde{k}} - e^{6\beta}\frac{k_x^2}{k_y^2}\frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}\lambda}\right)\mathcal{G}\mathcal{A}^* + (c.c.).
\end{aligned} \tag{A.20}$$

B Slow-roll expansion

B.1 Relation among small quantities in the slow-roll approximation

Since we are interested in the leading-order terms in the slow-roll expansion, we try to express small quantities in terms of the slow-roll parameters, ε_H and η_H , defined by

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2}, \quad \eta_H \equiv \frac{\ddot{H}}{2H\dot{H}}, \tag{B.1}$$

and the additional anisotropy parameter

$$I \equiv 2\frac{U}{U_\phi^2}\mathcal{V}, \quad \mathcal{V} \equiv \frac{f^2\dot{u}^2}{e^{2(\alpha-2\beta)}} = \frac{C_A^2}{f^2e^{4(\alpha+\beta)}}. \tag{B.2}$$

The equation for β reads

$$\ddot{\beta} + 3H\dot{\beta} = \frac{1}{3}\mathcal{V} \quad \rightarrow \quad \dot{\beta} \approx \frac{1}{3}e^{-3\alpha} \int dt e^{3\alpha} \left(\frac{1}{2} \frac{U_\phi^2}{U^2} \right) UI. \quad (\text{B.3})$$

The second Friedmann equation gives

$$\dot{H} + 3H^2 = U + \frac{1}{6}\mathcal{V} \quad \rightarrow \quad U = \frac{3 - \varepsilon_H}{1 + \frac{1}{12} \frac{U_\phi^2}{U^2} I} H^2. \quad (\text{B.4})$$

Combining this equation with the first Friedmann equation yields

$$\frac{1}{2} \frac{\dot{\phi}^2}{H^2} = \varepsilon_H - \left(\frac{1}{2} \frac{U_\phi^2}{U^2} \right) \left(1 - \frac{1}{3} \varepsilon_H \right) I + \mathcal{O}(I^2). \quad (\text{B.5})$$

The scalar field equation gives the following relation,

$$\ddot{\phi} + 3H\dot{\phi} + U_\phi(1 - cI) = 0 \quad \rightarrow \quad \sqrt{\frac{\frac{1}{2} \frac{\dot{\phi}^2}{H^2}}{\frac{1}{2} \frac{U_\phi^2}{U^2}}} = \frac{3 - \varepsilon_H}{3 + \frac{\dot{\phi}}{H\dot{\phi}}} \left[1 - \left(c + \frac{1}{12} \frac{U_\phi^2}{U^2} \right) I \right] + \mathcal{O}(I^2). \quad (\text{B.6})$$

Taking a time derivative of the second Friedmann equation gives

$$\eta_H = \frac{\frac{1}{2} \frac{\dot{\phi}^2}{H^2}}{\varepsilon_H} \frac{\ddot{\phi}}{H\dot{\phi}} + 2 \frac{\frac{1}{2} \frac{U_\phi^2}{U^2}}{\varepsilon_H} \left(1 - \frac{1}{3} \varepsilon_H \right) \left(c \sqrt{\frac{\frac{1}{2} \frac{\dot{\phi}^2}{H^2}}{\frac{1}{2} \frac{U_\phi^2}{U^2}}} - 1 \right) I + \mathcal{O}(I^2). \quad (\text{B.7})$$

Finally, by rewriting \dot{f}/f as

$$\frac{\dot{f}}{f} = \frac{f_\phi}{f} \dot{\phi} = 2c \frac{U}{U_\phi} \dot{\phi}, \quad (\text{B.8})$$

one finds the following expression,

$$\frac{\dot{f}}{f} = -2cH \left(1 - \frac{\varepsilon_H + \eta_H}{3} - \frac{c+2}{3} I \right) + \mathcal{O}(\varepsilon I, \varepsilon^2, I^2). \quad (\text{B.9})$$

B.2 Comparison with Watanabe, Kanno and Soda (2010)

By taking the logarithmic derivative of I in equation (B.2) with respect to t ,

$$\frac{\dot{I}}{I} = -\frac{\dot{\varepsilon}_H}{\varepsilon_H} - \frac{\dot{U}}{U} - 2\frac{\dot{f}}{f} - 4(H + \dot{\beta}), \quad (\text{B.10})$$

one obtains

$$\frac{\dot{I}}{I} = -2\frac{\dot{f}}{f} - 2H(2 + \eta_H) + \mathcal{O}(I), \quad (\text{B.11})$$

where we have assumed $U_\phi^2/(2U^2) \approx \varepsilon_H$.

Now, if we ignore \dot{I}/I , we have

$$\frac{\dot{f}}{f} = -H(2 + \eta_H) + \mathcal{O}(I). \quad (\text{B.12})$$

This equation coincides exactly with the formula derived in ref. [11]. (Note that their definition of the symbol ε_H is different from ours.) However, \dot{I}/I gives rise to terms of order ε_H , which cannot be ignored, and thus ignoring it leads to an erroneous result.

References

- [1] V. F. Mukhanov, H. Feldman, and R. H. Brandenberger, *Theory of cosmological perturbations*, *Phys.Rept.* **215** (1992) 203–333.
- [2] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuation and Nonsingular Universe*, *JETP Lett.* **33** (1981) 532–535.
- [3] **WMAP Collaboration**, G. Hinshaw et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, *Astrophys.J.Suppl.* **208** (2013) 19, [[arXiv:1212.5226](#)].
- [4] **Planck Collaboration** Collaboration, P. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, *Astron.Astrophys.* (2014) [[arXiv:1303.5076](#)].
- [5] **WMAP Collaboration**, C. Bennett et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results*, *Astrophys.J.Suppl.* **208** (2013) 20, [[arXiv:1212.5225](#)].
- [6] **Planck Collaboration** Collaboration, P. Ade et al., *Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity*, *Astron.Astrophys.* **571** (2014) A24, [[arXiv:1303.5084](#)].
- [7] E. Dimastrogiovanni, N. Bartolo, S. Matarrese, and A. Riotto, *Non-Gaussianity and Statistical Anisotropy from Vector Field Populated Inflationary Models*, *Adv.Astron.* **2010** (2010) 752670, [[arXiv:1001.4049](#)].
- [8] A. Maleknejad, M. Sheikh-Jabbari, and J. Soda, *Gauge Fields and Inflation*, *Phys.Rept.* **528** (2013) 161–261, [[arXiv:1212.2921](#)].
- [9] J. Soda, *Statistical Anisotropy from Anisotropic Inflation*, *Class.Quant.Grav.* **29** (2012) 083001, [[arXiv:1201.6434](#)].
- [10] L. Ackerman, S. M. Carroll, and M. B. Wise, *Imprints of a Primordial Preferred Direction on the Microwave Background*, *Phys.Rev.* **D75** (2007) 083502, [[astro-ph/0701357](#)].
- [11] M.-a. Watanabe, S. Kanno, and J. Soda, *The Nature of Primordial Fluctuations from Anisotropic Inflation*, *Prog.Theor.Phys.* **123** (2010) 1041–1068, [[arXiv:1003.0056](#)].
- [12] N. Barnaby, R. Namba, and M. Peloso, *Observable non-gaussianity from gauge field production in slow roll inflation, and a challenging connection with magnetogenesis*, *Phys.Rev.* **D85** (2012) 123523, [[arXiv:1202.1469](#)].
- [13] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, *The anisotropic power spectrum and bispectrum in the $f(\phi)F^2$ mechanism*, *Phys.Rev.* **D87** (2013) 023504, [[arXiv:1210.3257](#)].
- [14] M. Shiraishi, E. Komatsu, M. Peloso, and N. Barnaby, *Signatures of anisotropic sources in the squeezed-limit bispectrum of the cosmic microwave background*, *JCAP* **1305** (2013) 002, [[arXiv:1302.3056](#)].
- [15] M. Shiraishi, E. Komatsu, and M. Peloso, *Signatures of anisotropic sources in the trispectrum of the cosmic microwave background*, *JCAP* **1404** (2014) 027, [[arXiv:1312.5221](#)].
- [16] R. Emami and H. Firouzjahi, *Curvature Perturbations in Anisotropic Inflation with Symmetry Breaking*, *JCAP* **1310** (2013) 041, [[arXiv:1301.1219](#)].
- [17] A. A. Abolhasani, R. Emami, J. T. Firouzjaee, and H. Firouzjahi, *δN formalism in anisotropic inflation and large anisotropic bispectrum and trispectrum*, *JCAP* **1308** (2013) 016, [[arXiv:1302.6986](#)].
- [18] R. Emami, H. Firouzjahi, and M. Zarei, *Anisotropic Inflation with the non-Vacuum Initial State*, *Phys.Rev.* **D90** (2014) 023504, [[arXiv:1401.4406](#)].
- [19] A. Dey and S. Paban, *Non-Gaussianities in the Cosmological Perturbation Spectrum due to Primordial Anisotropy*, *JCAP* **1204** (2012) 039, [[arXiv:1106.5840](#)].

- [20] A. Dey, E. Kovetz, and S. Paban, *Non-Gaussianities in the Cosmological Perturbation Spectrum due to Primordial Anisotropy II*, *JCAP* **1210** (2012) 055, [[arXiv:1205.2758](#)].
- [21] A. Dey, E. D. Kovetz, and S. Paban, *Power Spectrum and Non-Gaussianities in Anisotropic Inflation*, *JCAP* **1406** (2014) 025, [[arXiv:1311.5606](#)].
- [22] S. Endlich, A. Nicolis, and J. Wang, *Solid Inflation*, *JCAP* **1310** (2013) 011, [[arXiv:1210.0569](#)].
- [23] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, *Anisotropy in solid inflation*, *JCAP* **1308** (2013) 022, [[arXiv:1306.4160](#)].
- [24] N. Bartolo, M. Peloso, A. Ricciardone, and C. Unal, *The expected anisotropy in solid inflation*, *JCAP* **1411** (2014) 009, [[arXiv:1407.8053](#)].
- [25] M. Akhshik, R. Emami, H. Firouzjahi, and Y. Wang, *Statistical Anisotropies in Gravitational Waves in Solid Inflation*, *JCAP* **1409** (2014) 012, [[arXiv:1405.4179](#)].
- [26] M.-a. Watanabe, S. Kanno, and J. Soda, *Inflationary Universe with Anisotropic Hair*, *Phys.Rev.Lett.* **102** (2009) 191302, [[arXiv:0902.2833](#)].
- [27] J. Kim and E. Komatsu, *Limits on anisotropic inflation from the Planck data*, *Phys.Rev.* **D88** (2013) 101301, [[arXiv:1310.1605](#)].
- [28] P. Creminelli, J. Norena, and M. Simonovic, *Conformal consistency relations for single-field inflation*, *JCAP* **1207** (2012) 052, [[arXiv:1203.4595](#)].
- [29] A. Maleknejad and M. Sheikh-Jabbari, *Revisiting Cosmic No-Hair Theorem for Inflationary Settings*, *Phys.Rev.* **D85** (2012) 123508, [[arXiv:1203.0219](#)].
- [30] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, *The Effective Field Theory of Inflation*, *JHEP* **0803** (2008) 014, [[arXiv:0709.0293](#)].
- [31] D. Cannone, G. Tasinato, and D. Wands, *Generalised tensor fluctuations and inflation*, [arXiv:1409.6568](#).